Reduced density matrices
and Bose-Einstein condensation

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6 June 2007

Abstract
Emergence and applications of the ubiquitous tool of reduced density matrices in the rigorous analysis of Bose Einstein condensation is reviewed, and new related results are added.
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